

7.1 ★ Write down the Lagrangian for a projectile (subject to no air resistance) in terms of its Cartesian coordinates (x, y, z) , with z measured vertically upward. Find the three Lagrange equations and show that they are exactly what you would expect for the equations of motion.

7.2 ★ Write down the Lagrangian for a one-dimensional particle moving along the x axis and subject to a force $F = -kx$ (with k positive). Find the Lagrange equation of motion and solve it.

7.4 ★ Consider a mass m moving in a frictionless plane that slopes at an angle α with the horizontal. Write down the Lagrangian in terms of coordinates x , measured horizontally across the slope, and y , measured down the slope. (Treat the system as two-dimensional, but include the gravitational potential energy.) Find the two Lagrange equations and show that they are what you should have expected.

7.8 ★★ (a) Write down the Lagrangian $\mathcal{L}(x_1, x_2, \dot{x}_1, \dot{x}_2)$ for two particles of equal masses, $m_1 = m_2 = m$, confined to the x axis and connected by a spring with potential energy $U = \frac{1}{2}kx^2$. [Here x is the extension of the spring, $x = (x_1 + x_2 - l)$, where l is the spring's unstretched length, and I assume that mass 1 remains to the right of mass 2 at all times.] (b) Rewrite \mathcal{L} in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ (the CM position) and x (the extension), and write down the two Lagrange equations for X and x . (c) Solve for $X(t)$ and $x(t)$ and describe the motion.

6-9. Find an expression involving the function $\phi(x_1, x_2, x_3)$ that has a minimum average value of the square of its gradient within a certain volume V of space.

6-10. Find the ratio of the radius R to the height H of a right-circular cylinder of fixed volume V that minimizes the surface area A .

6-14. Find the shortest path between the (x, y, z) points $(0, -1, 0)$ and $(0, 1, 0)$ on the conical surface $z = 1 - \sqrt{x^2 + y^2}$. What is the length of the path? Note: this is the shortest mountain path around a volcano.